2025 East Asia Workshop on Extremal and Structural Graph Theory

School of Mathematics

Sun Yat-sen University

Guangzhou, China

March 27-30, 2025

2025 East Asia Workshop on Extremal and Structural Graph Theory

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Venue

Room 209, New Math. Building, Sun Yat-sen University

No. 135, Xingang Xi Road, Haizhu District, Guangzhou, China

Organizing Committee

Ping Hu	Sun Yat-sen University, China
Seog-jin Kim	Konkuk University, South Korea
Kenta Ozeki	Yokohama National University, Japan
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Program

Day 0	March 27	
14:00-21:00	Registration (Sun Yat-sen Kaifeng Hotel)	
18:00-20:00	Dinner (Sun Yat-sen Kaifeng Hotel)	
Day 1	**March 28**	
09:20-09:50	Chair: Ping Hu	Zixiang Xu Dense graphs without certain subgraphs: structural insights and thresholds
10:00-10:30		Joonkyung Lee On norming systems of linear equations
10:30-10:50	Coffee Break	
10:50-11:20	Chair: Atsuhiro Nakamoto	Kenta Ozeki Colouring normal quadrangulations of projective spaces
11:30-12:00		Zhouningxin Wang Strongly-connected orientations and circular flow in signed graphs
12:00-14:00	Lunch (Zijingyuan Hotel 紫荆园宾馆)	
14:20-14:50	Chair: Seog-Jin Kim	Chong Shangguan Tree packings with applications to coding theory
15:00-15:30		Ilkyoo Choi An improved lower bound on the number of edges in list critical graphs via DP-coloring
15:30-15:50	Coffee Break	
15:50-17:30	Problem Session	
18:00-20:00	Dinner (Zijingyuan Hotel 紫荆园宾馆)	
Day 2	**March 29**	
09:20-09:50	Chair: Bo Ning	Hikaru Yokoi Some problems on path-chromatic number
10:00-10:30		Masaki Kashima A relaxation of the Chvátal-Erdős condition and 2-factors
10:30-10:50	Group Photo, Coffee Break	
10:50-11:20	Chair: Tuan Tran	Longtu Yuan Stability in Bondy's theorem on paths and cycles
11:30-12:00		Laihao Ding Hypergraphs with arbitrarily small 2-degree Turán density
12:00-14:00	Lunch (Zijingyuan Hotel 紫荆园宾馆)	
14:20-14:50	Chair: Jongyook Park	Hyunwoo Lee A quantitative improvement on the hypergraph Balog–Szemerédi–Gowers theorem
15:00-15:30		Seonghyuk Im Ramsey–Dirac theory for bounded degree hypertrees

15:40-16:10		Shenwei Huang Critical (P_6, banner)-free graphs
16:10-16:30	Coffee Break	
16:30-17:30	Problem Session	
18:00-20:00	Dinner (Zijingyuan Hotel 紫荆园宾馆)	
Day 3	**March 30**	
09:20-09:50	Chair: Binlong Li	Jaehyeon Seo Graph homomorphism inequalities and Lorentzian polynomials
10:00-10:30		Ruonan Li Sidorenko's conjecture for subdivisions and theta substitutions
10:30-10:50	Coffee Break	
10:50-11:20	Chair: Hehui Wu	Marton Marits Cover numbers by certain graph families
11:30-12:00		On-Hei Solomon Lo Minors of non-hamiltonian graphs
12:00-14:00	Lunch (Zijingyuan Hotel 紫荆园宾馆)	
14:20-17:30	Problem Session	
18:00-20:00	Dinner (Zijingyuan Hotel 紫荆园宾馆)	

An improved lower bound on the number of edges in list critical graphs via DP-coloring

Ilkyoo Choi
Hankuk University of Foreign Studies, South Korea

A graph G is (list, DP) k-critical if the (list, DP) chromatic number is k but for every proper subgraph G' of G, the (list) chromatic number of G' is less than k. For $k \geq 4$, we show a bound on the minimum number of edges in a DP k-critical graph, and our bound is the first bound that is asymptotically better than the corresponding bound for proper k-critical graphs by Gallai from 1963. Our result also improves the best bound on the list chromatic number.

This is joint work with Peter Bradshaw (University of Illinois at Urbana-Champaign), Alexandr V. Kostochka (University of Illinois at Urbana-Champaign), and Jingwei Xu (University of Illinois at Urbana-Champaign).

Hypergraphs with arbitrarily small 2-degree Turán density

Laihao Ding
Central China Normal University, China

For a k-uniform hypergraph (or simply k-graph) F, the 2-degree Turán density $\pi_2(F)$ is the supremum over all α such that there exist infinitely many F-free k-graphs H such that every pair of $V(H)$ is contained in at least $\alpha\binom{n}{k-2}$ edges. By relating to the vanishing order, which was first introduced by Reiher, Rödl and Schacht in the study of vanishing uniform Turán density, we show that the 2-degree Turán density can be arbitrarily close to zero. This is a joint work with Hong Liu and Haotian Yang.

Critical $(P_6, banner)$-free graphs

Shenwei Hunag
Nankai University, China

In this talk, we prove that for every fixed positive integer k, there are finitely many k-critical $(P_6, banner)$-free graphs. This is one of the few results on k-critical graphs restricted to P_6-free graphs and generalizes a known result by Hell and Huang.

Ramsey–Dirac theory for bounded degree hypertrees

Seonghyuk Im
KAIST/IBS ECOPRO, South Korea

Ramsey–Turán theory considers Turán type questions in Ramsey-context, asking for the existence of a small subgraph in a graph G where the complement $\overline{G}$ lacks an appropriate subgraph F, such as a clique of linear size. Similarly, one can consider Dirac-type questions in Ramsey context, asking for the existence of a spanning subgraph H in a graph G where the complement $\overline{G}$ lacks an appropriate subgraph F, which we call a Ramsey–Dirac theory question.

When H is a connected spanning subgraph, the disjoint union $K_{n/2} \cup K_{n/2}$ of two large cliques shows that it is natural to consider complete bipartite graphs F. Indeed, Han, Hu, Ping, Wang, Wang and Yang in 2024 proved that if G is an n-vertex graph with $\delta(G) = \Omega(n)$ where the complement $\overline{G}$ does not contain any complete bipartite graph $K_{m,m}$ with $m = \Omega(n)$, then G contains every n-vertex bounded degree tree T as a subgraph.

Extending this result to the Ramsey–Dirac theory for hypertrees, we prove that if G is an n-vertex r-uniform hypergraph with $\delta(G) = \Omega(n^{r-1})$ where the complement $\overline{G}$ does not contain any complete r-partite hypergraph $K^{(r)}_{m,m,\dots,m}$ with $m = \Omega(n)$, then G contains every n-vertex bounded degree hypertree T as a subgraph. We also prove the existence of matchings and loose Hamilton cycles in the same setting, which extends the result of Mcdiarmid and Yolov into hypergraphs.

The lack of $K_{m,m,\dots,m}$ in $\overline{G}$ can be viewed as a very weak form of pseudorandomness condition on the hypergraph G. Hence, our results have some interesting implications on pseudorandom hypgraphs. For examples, they generalize the universality result on randomly perturbed graphs by Böttcher, Han, Kohayakawa, Montgomery, Parczyk and Person in 2019 into hypergraphs and also strengthen the results on quasirandom hypergraphs by Lenz, Mubayi and Mycroft in 2016 and Lenz and Mubayi in 2016 into hypergraphs satisfying a much weaker pseudorandomness condition.

This is a joint work with Jie Han (BIT), Jaehoon Kim (KAIST), and Donglei Yang (Shandong University).

A relaxation of the Chvátal-Erdős condition and 2-factors

Masaki Kashima
Keio University, Japan

A Haimlton cycle of a graph is a cycle passing through all the vertices and a 2-factor of a graph is a 2-regular spanning subgraph. In 1972, Chvátal and Erdős showed that any graph G with $\alpha(G) \leq \kappa(G)$ has a Hamilton cycle, where $\alpha(G)$ is the independence number and $\kappa(G)$ is the connectivity of G. Since $\kappa(G)$ is at most the minimum degree for any graph G, we consider Hamiltonicity of a graph G with $\alpha(G) \leq \delta(G)$, which is a relaxation of the Chvátal-Erdős condition. There are disconnected graph G with $\alpha(G) \leq \delta(G)$, and hence the condition $\alpha(G) \leq \delta(G)$ does not imply the existence of a Hamilton cycle. On the other hand, we can show that (1) every graph G with $\alpha(G) \leq \delta(G) - 1$ has a 2-factor and that (2) every graph G with $\alpha(G) \leq \delta(G)$ has a 2-factor unless G belongs to a complete determined exceptional graphs. In this talk, we discuss these topics together with results on additional conditions for a graph G with $\alpha(G) \leq \delta(G) - 1$ to have a 2-factor with at most k cycles.

A quantitative improvement on the hypergraph Balog–Szemerédi–Gowers theorem

Hyunwoo Lee
KAIST / IBS ECOPRO, South Korea

In this note, we obtain a quantitative improvement on the hypergraph variant of the Balog–Szemerédi–Gowers theorem due to Sudakov, Szemerédi, and Vu [Duke Math. J.129.1 (2005): 129–155]. Additionally, we prove the hypergraph variant of the “almost all” version of Balog–Szemerédi–Gowers theorem.

On norming systems of linear equations

Joonkyung Lee
Yonsei University, South Korea

A system of linear equations L is said to be norming if a natural functional $t_L(\cdot)$ giving a weighted count for the set of solutions to the system can be used to define a norm on the space of real-valued functions on $\mathbb{F}_q^n$ for every $n > 0$. For example, Gowers uniformity norms arise in this way. We initiate the systematic study of norming linear systems by proving a range of necessary and sufficient conditions for a system to be norming. Some highlights include an isomorphism theorem for the functional $t_L(\cdot)$, a proof that any norming system must be variable-transitive and the classification of all norming systems of rank at most two.

This is joint work with Seokjoon Cho (Seoul National University), David Conlon (Caltech), Jozef Skokan (LSE) and Leo Versteegen (LSE).

Sidorenko's conjecture for subdivisions and theta substitutions

Ruonan Li
Northwestern Polytechnical University, China

The famous Sidorenko's conjecture asserts that for every bipartite graph H, the number of homomorphisms from H to a graph G with given edge density is minimized when G is pseudorandom. We prove that for any graph H, a graph obtained from replacing edges of H by generalized theta graphs consisting of even paths satisfies Sidorenko's conjecture, provided a certain divisibility condition on the number of paths. To achieve this, we prove unconditionally that bipartite graphs obtained from replacing each edge of a complete graph with a generalized theta graph satisfy Sidorenko's conjecture, which extends a result of Conlon, Kim, Lee and Lee [J. Lond. Math. Soc., 2018]. This is a joint work with Seonghyuk Im and Hong Liu.

Minors of non-hamiltonian graphs

On-Hei Solomon Lo
Tongji University, China

A seminal result of Tutte asserts that every 4-connected planar graph is hamiltonian. By Wagner's theorem, Tutte's result can be restated as: every 4-connected graph without a $K_{3,3}$ minor is hamiltonian. Recently, Ding and Marshall asked for a characterization of the minor-minimal 3-connected non-hamiltonian graphs. They conjecture that every 3-connected non-hamiltonian graph contains a minor of $K_{3,4}$, $\mathfrak{Q}^+$, or the Herschel graph, where $\mathfrak{Q}^+$ is obtained from the cube by adding a new vertex and joining it to three vertices that share a common neighbor in the cube. In this talk, we discuss this conjecture and related problems.

Cover numbers by certain graph families

Marton Marits
Yokohama National University, Japan

Let G be a graph, and P a family of graphs. The cover number of G by P is the smallest number of P-graphs needed to cover G. We survey some classical results about some choices of P where the cover number is known exactly, as well as some recent extensions to these theorems. We then find some interesting results about the cover number by comparability graphs.

Colouring normal quadrangulations of projective spaces

Kenta Ozeki
Yokohama National University, Japan

Youngs proved that every non-bipartite quadrangulation of the projective plane $\mathbb{RP}^2$ is 4-chromatic. Hachimori et al. defined a high-dimensional quadrangulation, called a normal quadrangulation. They proved that if a non-bipartite normal quadrangulation G of the d-dimensional projective space $\mathbb{RP}^d$ with any $d \geq 2$ satisfies a certain geometric condition, then G is 4-chromatic, and asked whether the geometric condition can be removed from the result. In this talk, we give a negative solution to their problem for the case $d = 3$, proving that there exist 3-dimensional normal quadrangulations of $\mathbb{RP}^3$ whose chromatic number is arbitrarily large. Moreover, we prove that no normal quadrangulation of $\mathbb{RP}^d$ with any $d \geq 2$ has chromatic number.

This is a jointwork with Tomáš Kaiser (University of West Bohemia in Pilsen, Czech Republic), On-Hei Solomon Lo (Tongji University, China), Atsuhiro Nakamoto (Yokohama National University, Japan) and Yuta Nozaki (Yokohama National University and Hiroshima University, Japan).

Graph homomorphism inequalities and Lorentzian polynomials

Jaehyeon Seo
Yonsei University, South Korea

For graphs H and G, a homomorphism from H to G is a map between their vertex sets which preserves adjacency. We focus on homomorphisms into antiferromagnetic graphs G, meaning G has nonnegative weights, at most one positive eigenvalue, and possibly contains loops. The number of homomorphisms into such graphs is important as they generalize various fundamental graph parameters, including the number of independent sets and proper colorings.

We prove a number of inequalities on the number of homomorphisms into antiferromagnetic targets, and as a consequence provide infinitely many instances that support a conjecture of Sah, Sawhney, Stoner, and Zhao. Our method leverages the emerging theory of Lorentzian polynomials due to Brändén and Huh, and represents the first use of this powerful framework in extremal graph theory.

This is joint work with Joonkyung Lee and Jaeseong Oh.

Tree packings with applications to coding theory

Chong Shangguan
Shandong University, China

Given a matrix and a set S of its columns, a row is said to *separate* S if, restricted to this row, these columns have distinct values. We show that given integers $1 \leq k < n$ and $t \geq 3$, for a sufficiently large prime power q, there exists an $n \times q^k$ matrix, defined on the alphabet $\mathbb{F}_q$, such that any set of t columns is separated by at least $n - k(t-1) + 1$ rows. (The value $n - k(t-1) + 1$ is conjectured to be optimal.) We prove the above result by a surprising connection to graph theory, and in particular to the cycle space of complete graphs and the tree packing theorem of Nash-Williams and Tutte. We also state a new conjecture that generalizes the tree-packing theorem to hypergraphs. If this conjecture holds, then there would exist Reed-Solomon codes that are optimally list-decodable.
References: Zeyu Guo, Ray Li, Chong Shangguan, Itzhak Tamo, Mary Wootters, Improved list-decoding of Reed-Solomon codes via tree packings, SIAM J. Comput. 53 (2024), no. 2, 389–430.

Strongly-connected orientations and circular flow in signed graphs

Zhouningxin Wang
Nankai University, China

In this talk, we explore a relation between modular $2k$-orientations achieving boundary conditions and circular flows in signed graphs. A mapping $\beta : V(G) \to \{0, \pm 1, \ldots, \pm k\}$ is said to be a *parity-compliant* $\mathbb{Z}_{2k}$*-boundary* of G if for every vertex $v \in V(G)$, $\beta(v) \equiv d(v) \pmod 2$ and $\sum_{v \in V(G)} \beta(v) \equiv 0 \pmod{2k}$. We shall prove that for $3 \leq k \leq 7$, if a planar graph G is $(3k-1)$-edge-connected, then for any parity-compliant $\mathbb{Z}_{2k}$-boundary β on $2G$, $2G$ admits a strongly-connected orientation D such that $d^+_D(v) - d^-_D(v) \equiv \beta(v) \pmod{2k}$. In particular, it implies that every signed planar graph of girth at least 8 has its circular flow index strictly less than 3. This is joint work with J. Li, Z. Miao, Y. Shi, and C. Wei.

Dense graphs without certain subgraphs: structural insights and thresholds

Zixiang Xu
Institute for Basic Science, South Korea

Understanding the structural properties of dense graphs that exclude certain subgraphs remains one of the central challenges in extremal graph theory. A fundamental yet notoriously difficult problem in this direction is to determine the optimal minimum degree condition under which every large H-free graph admits a homomorphism into an H-free graph of constant size, for a given graph H.

In this talk, I will present a framework we recently developed to tackle this problem, which is mainly based on the theory of VC-dimension. Our approach not only unifies and extends several classical results by Thomassen, Łuczak and Thomassé, and Goddard, Lyle and Nikiforov, but also strengthens recent results of Ebsen, Oberkampf, and Schacht. The results presented in this talk are based on joint works with Xinqi Huang, Hong Liu, Mingyuan Rong, Jozef Skokan and Chong Shangguan.

Some problems on path-chromatic number

Hikaru Yokoi
Keio University, Japan

The *chromatic number of a tree-decomposition* of a graph is the maximum chromatic number among the subgraphs induced by its bag. Seymour (2016) defined the *tree-chromatic number* of a graph G to be the minimum chromatic number of the tree-decompositions of G. If we only consider the path-decompositions of G, the *path-chromatic number* of G is analogously defined. Huynh, Reed, Wood and Yepremyan (2021) surveyed tree- and path-chromatic numbers of graphs, and posed some interesting problems including Hadwiger-type conjectures and decision problems. In this talk, we present our results inspired by their work and pose new problems.

Stability in Bondy's theorem on paths and cycles

Longtu Yuan
East China Normal University, China

In this paper, we study the stability result of a well-known theorem of Bondy. We prove that for any 2-connected non-hamiltonian graph, if every vertex except for at most one vertex has degree at least k, then it contains a cycle of length at least $2k+2$ except for some special families of graphs. Our results imply several previous classical theorems including a deep and old result by Voss. We point out our result on stability in Bondy's theorem can directly imply a positive solution (in a slight stronger form) to the following problem: Is there a polynomial time algorithm to decide whether a 2-connected graph G on n vertices has a cycle of length at least $\min\{2\delta(G)+2, n\}$? This problem originally motivates the recent study on algorithmic aspects of Dirac's theorem by Fomin, Golovach, Sagunov, and Simonov, although a stronger problem was solved by them by completely different methods. Our theorem can also help us to determine all extremal graphs for wheels on odd number of vertices. We also discuss the relationship between our results and some previous problems and theorems in spectral graph theory and generalized Turán problems.