# The 3rd East Asia Workshop on Extremal and Structural Graph Theory

**Date: 1 (Wednesday)–5 (Sunday), November 2023**

**Venue： The Southern Beach Hotel & Resort Okinawa**
**1-6-1, Nishizakicho, Itoman-city, Okinawa, 901-0306, Japan**

## DAY 0 – November 1st (Wed.)

16:00 ∼ 18:00 Registration and Discussions

## DAY 1 – November 2nd (Thurs.)

9:20 ∼ 9:50 Kiyoshi Yoshimoto (Nihon University, Japan)
*On Connectivities of Edge-Colored Graphs*

10:00 ∼ 10:30 Dong-Yeap Kang (IBS, Korea)
*Robust Hamiltonicity of Dirac hypergraphs*

——— Coffee Break ———

10:50 ∼ 11:20 Jiangdong Ai (Nankai University, China)
*Selected Conjectures and Open Problems on Directed Graphs*

11:30 ∼ 12:00 Problem Session

——— Lunch ———

14:00 ∼ 14:30 Bill Jackson (Queen Mary University of London, UK)
*Rigid Graphs*

14:40 ∼ 15:10 Linda Cook (IBS, Korea)
*Reuniting Chi-Boundedness and Polynomial Chi-Boundedness*

——— Coffee Break ———

15:30 ∼ 17:00 Problem Session

# DAY 2 – November 3rd (Fri.)

9:20 〜 9:50 Shinya Fujita (Yokohama City University, Japan)
*Safe set problems in vertex-weighted graphs and digraphs*

10:00 〜 10:30 Ilkyoo Choi (Hankuk University of Foreign Studies, Korea)
*Between proper coloring graphs and squares of graphs*

——— Coffee Break ———

10:50 〜 11:20 Xujun Liu (Xijiao-Liverpool University, China)
*Every subcubic multigraph is* $(1, 2^7)$*-packing edge-colorable*

11:30 〜 12:00 Problem Session

——— Lunch ———

14:00 〜 14:30 Shunichi Maezawa (Tokyo University of Science, Japan)
*Forbidden immersion condition for graphs to be* $7$*-colorable*

14:40 〜 15:10 Shagnik Das (National Taiwan University, Taiwan)
*Explicit constructions of strong blocking sets and minimal codes*

——— Coffee Break ———

15:30 〜 17:00 Problem Session

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18:00 〜 20:00 Banquet

## DAY 3 – November 4 (Sat.)

9:00 Open the desk

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9:20 ~ 9:50 Henry Liu (Sun Yet-Sen University, China)
*Connected subgraphs in coloured graphs: recent developments*

10:00 ~ 10:30 Joonkyung Lee (Yonsei University, Korea)
*Extremal numbers and Sidorenko's conjecture*

— Coffee Break —

10:50 ~ 11:20 Shoichi Tsuchiya (Senshu University, Japan)
*Degree conditions for the existence of a homeomorphically irreducible spanning tree*

11:30 ~ 12:00 Tuan Tran (University of Science and Technolgy, China)
*Complexity of null dynamical systems and Sauer-Shelah lemmas*

— Lunch —

14:00 ~ 17:00 Discussions

## DAY 4 – November 5 (Sun.)

9:00 ~ 11:00 Discussions

**Abstracts:**

## On Connectivities of Edge-Colored Graphs

Kiyoshi Yoshimoto

Nihon University, Japan

Let $(G, c)$ be an edge-colored graph where $c : E(G) \to \mathbb{N}$ is an edge-coloring. For $x, y \in V(G)$, if there exists a sequence of properly colored cycles $C_1, C_2, \ldots, C_l$ in $G$ such that $x \in V(C_1), y \in V(C_l)$ and $V(C_i) \cap V(C_{i+1}) \neq \emptyset$ for any $1 \leq i < l$, then we say the pair $x, y$ is *cyclically color-connected*, denoted $x \approx y$. The color of the edge incident to an end vertex $x$ of a path $P$ is denoted by $c_x(P)$. If there exist two properly colored paths $P, Q$ joining $x$ and $y$ such that $c_x(P) \neq c_x(Q)$ and $c_y(P) \neq c_y(Q)$, then the pair $x, y$ is called *color-connected*, denoted $x \sim y$. If color-connectivity satisfies a transitive relation on $V(G)$, then we say $(G, c)$ is *convenient*. It is straightforward to see that if $x \approx y$, then always $x \sim y$. However the converse does not hold always. An edge-colored graph $(G, c)$ is called *strongly convenient* if the converse holds. Saad showed that 2- edge-colored complete graphs are convenient. Bang-Jensen and Gutin generalized it for some family of 2-edge-colored complete multipartite graphs. In this talk, we discuss an extension those results.

## Robust Hamiltonicity of Dirac hypergraphs

Dong-Yeap Kang

IBS ECOPRO, Korea

We prove new robustness results for Hamilton $\ell$-cycles in Dirac hypergraphs under *exact* minimum codegree conditions.

Given a hypergraph $\mathcal{H}$, let $\mathcal{H}_p$ be a spanning subgraph of $\mathcal{H}$ obtained by taking each edge of $\mathcal{H}$ independently with probability $p$.

Let $\mathcal{H}$ be a $k$-uniform hypergraph on $n$ vertices with minimum codegree at least $\frac{n}{2(k-\ell)}$ and $(k-\ell) \mid n$. Then we prove the following.

(i) If $p = \Omega(\log n/n^{k-1})$, then a.a.s. $\mathcal{H}_p$ contains a loose Hamilton cycle when $\ell = 1$.

(ii) If $p = \omega(1/n^{k-\ell})$, then a.a.s. $\mathcal{H}_p$ contains a Hamilton $\ell$-cycle for all $2 \leq \ell < k/2$.

The bound on the minimum codegree is tight, and the bounds on $p$ are also best possible up to a constant factor.

In particular, we prove the results in terms of 'vertex-spreadness' introduced recently by Kelly, Müyesser, and Pokrovskiy, answering their question when $1 \leq \ell < k/2$.

This is joint work with Michael Anastos, Debsoumya Chakraborti, Abhishek Methuku, and Vincent Pfenninger.

## Selected Conjectures and Open Problems on Directed Graphs

Jiangdong Ai
Nankai University, China

Digraph theory has a large number of conjectures and open problems which are easy to state but hard to solve. Many of these conjectures and problems were posed quite a while ago. Apart from providing discussion of known results on already published conjectures and open problems, we pose new conjectures and open problems based on some (our) rescent results.

## Rigid Graphs

Bill Jackson
School of Mathematical Sciences, Queen Mary University of London, UK

A graph $G$ is *rigid in* $\mathbb{R}^d$ if, for any generic realisation of $G$ in $\mathbb{R}^d$, every continuous motion of the vertices which preserves the distances between all pairs of adjacent vertices, preserves the distances between all pairs of vertices. It is straightforward to show that $G$ is rigid in $\mathbb{R}^1$ if and only if it is connected. Rigidity in $\mathbb{R}^2$ is characterized by a celebrated result of Pollaczek-Geiringer (1927), subsequently rediscovered by Laman (1970). Extending this characterization to $\mathbb{R}^3$ is an important open problem in discrete geometry.

After a brief introduction to graph rigidity, I will describe a recent result with Katie Clinch and Sin-ichi Tanigawa (2022) which characterises the closely related property of $C_1^2$*-rigidity* (which is conjectured to be equivalent to rigidity in $\mathbb{R}^3$).

This is a joint work with Katie Clinch (University of Sydney) and Shin-ichi Tanigawa (University of Tokyo).

## Reuniting Chi-Boundedness and Polynomial Chi-Boundedness

Linda Cook
IBS DIMAG, Korea

A class F of graphs is chi-bounded if there is a function f such that chi(H) is at most f( ω (H)) for all induced subgraphs H of a graph in F. If f can be chosen to be a polynomial, we say that F is polynomially chi-bounded. Esperet proposed a conjecture that every chi-bounded class of graphs is polynomially chi-bounded. This conjecture has been disproved; it has been shown that there are classes of graphs that are chi-bounded but not polynomially chi-bounded. As an attempt to understand the structural distinctions between chi-bounded graph classes and polynomially chi-bounded graph classes, we introduce Pollyanna classes of graphs. A class C of graphs is Pollyanna if the intersection of C and F is polynomially chi-bounded for every chi-bounded class F of graphs. We prove that several classes of graphs are Pollyanna and also present a non-trivial class of graphs that is not Pollyanna. (https://arxiv.org/abs/2310.11167) Joint work with Maria Chudnovsky, James Davies and Sang-il Oum

## Safe set problems in vertex-weighted graphs and digraphs

Shinya Fujita

School of Data Science, Yokohama City University, Japan

A non-empty subset $S$ of the vertices of a connected graph $G$ is a *safe set* if, for every connected component $C$ of $G[S]$ and every connected component $D$ of $G - S$, we have $|C| \geq |D|$ whenever there exists an edge of $G$ between $C$ and $D$. Since I introduced the concept of the safe set in a graph in [S. Fujita, G. MacGillivray, T. Sakuma: Safe set problem on graphs. Discret. Appl. Math. 215: 106-111 (2016)], this idea has garnered attention and various related results have been demonstrated.

To commemorate this joint research symposium with China and South Korea, I would like to present our recent research findings in this topic with our collaborative researchers from China and South Korea. More precisely, some recent results on safe set problems in vertex-weighted graphs and digraphs will be reviewed from the following joint papers.

### References

[1] R. Águeda, N. Cohen, S. Fujita, S. Legay, Y. Manoussakis, Y. Matsui, L. Montero, R. Naserasr, H. Ono, Y. Otachi, T. Sakuma, Z. Tuza, R. Xu, Safe sets in graphs: Graph classes and structural parameters. *Journal of Combinatorial Optimization*, **36** (2018) 1221-1242.

[2] Y. Bai, J. Bang-Jensen, S. Fujita, H. Ono, A. Yeo, Safe sets and in-dominating sets in digraphs. *Discrete Applied Mathematics*, to appear.

[3] S. Fujita, B. Park, T. Sakuma, Stable structure on safe set problems in vertex-weighted graphs. *European Journal of Combinatorics*, **91**(2021) #103211.

[4] S. Fujita, B. Park, T. Sakuma, Stable Structure on safe set problems in vertex-weighted graphs II -Recognition and complexity-. *Lecture Notes in Computer Science (WG 2020)*, **12301** 364-375.

## Between proper coloring graphs and squares of graphs

ILKYOO CHOI

Hankuk University of Foreign Studies, Korea

The following two notions of relaxations of coloring squares of graphs were recently formally introduced by Petruševski and Škrekovski, and by Fabrici, Lužar, Rindošová, and Soták: An odd c-coloring (resp. proper conflict-free c-coloring) of a graph is a proper c-coloring such that each non-isolated vertex has a color appearing an odd number of times (resp. exactly once) on its neighborhood. Since its appearance on the arXiv, there has been over 20 papers on these coloring notions. In this talk, we survey some of these results and conjectures, which includes Brooks-type results and strengthening of the Four Color Conjecture, and so on.

## Every subcubic multigraph is $(1, 2^7)$-packing edge-colorable

Xujun Liu

School of Mathematical Sciences, Xi'an Liverpool University, China

For a non-decreasing sequence $S = (s_1, \ldots, s_k)$ of positive integers, an $S$-packing edge-coloring of a graph $G$ is a decomposition of edges of $G$ into disjoint sets $E_1, \ldots, E_k$ such that for each $1 \leq i \leq k$ the distance between any two distinct edges $e_1, e_2 \in E_i$ is at least $s_i + 1$. The notion of $S$-packing edge-coloring was first generalized by Gastineau and Togni from its vertex counterpart. They showed that there are subcubic graphs that are not $(1, 2, 2, 2, 2, 2, 2)$-packing (abbreviated to $(1, 2^6)$-packing) edge-colorable and asked the question whether every subcubic graph is $(1, 2^7)$-packing edge-colorable. Very recently, Hocquard, Lajou, and Lužar showed that every subcubic graph is $(1, 2^8)$-packing edge-colorable and every 3-edge colorable subcubic graph is $(1, 2^7)$-packing edge-colorable. Furthermore, they also conjectured that every subcubic graph is $(1, 2^7)$-packing edge-colorable.

In this paper, we confirm the conjecture of Hocquard, Lajou, and Lužar, and extend it to multigraphs. This is a joint work with Santana and Short.

## Forbidden immersion condition for graphs to be 7-colorable

Shunichi Maezawa

Tokyo University of Science, Japan

A graph $H$ is an *immersion* of a graph $G$ if there exist functions $f_1 : V(H) \to V(G)$ and $f_2$ mapping from the edges of $H$ to paths of $G$ satisfying that

- $f_1$ is an injection,
- for $uv \in E(H)$, $f_2(uv)$ is a path connecting $f_1(u)$ and $f_2(v)$, and
- for edges $e, e' \in E(H)$ with $e \neq e'$, $f_2(e)$ and $f_2(e')$ are pairwise edge-disjoint.

Similar to a property of a minor, if $G$ has a subdivision of $H$, then $G$ has $H$ as an immersion but the converse is not true. However, the existence of an $H$ minor and an $H$ immersion are incomparable. It would be interesting to investigate relations between the chromatic number and a complete graph immersion, as in Hadwiger's conjecture. In fact, Abu-Khzam and Langston posed the following conjecture in [1].

**Conjecture 1** Every graph with no $K_t$ as an immersion is $(t-1)$-colorable.

Lescure and Meyniel [2] proved the conjecture for $t = 5, 6$ and DeVos, Kawarabayashi, Mohar, and Okamura [3] proved the conjecture for $t = 7$. We show that every graph with no $K_8^-$ and $R$ as immersions is 7-colorable. The graph $R$ will be defined in the talk.

## References

[1] F. N. Abu-Khzam and M. A. Langston, Graph coloring and the immersion order, in: Computing and Combinatorics, in: Lecture Notes in Comput. Sci., vol. 2697. Springer, Berlin, 2003, pp. 394–403.

[2] F. Lescure and H. Meyniel, On a problem upon configurations contained in graphs with given chromatic number, Graph theory in memory of G. A. Dirac, 325–331, Ann. Discrete Math., 41, North-Holland, Amsterdam, 1989.

[3] M. DeVos, K. Kawarabayashi, B. Mohar, and H. Okamura, Immersing small complete graphs, *Ars. Math. Contemp.* **3** (2) (2010) 139–146.

## Explicit constructions of strong blocking sets and minimal codes

Shagnik Das

National Taiwan University, Taiwan

A strong blocking set in a finite projective space is a set of points that intersects each hyperplane in a spanning set. In this talk we present a new graph theoretic construction of such sets: combining constant-degree expanders with asymptotically good codes, we explicitly construct strong blocking sets in the $(k-1)$-dimensional projective space over $\mathbb{F}_q$ that have size $O(qk)$. Since strong blocking sets have recently been shown to be equivalent to minimal linear codes, our construction gives the first explicit construction of $\mathbb{F}_q$-linear minimal codes of length $n$ and dimension $k$, for every prime power $q$, for which $n = O(qk)$.

This is joint work with Noga Alon, Anurag Bishnoi and Alessandro Neri.

## Connected subgraphs in coloured graphs: recent developments

Henry Liu

Sun Yat-sen University, China

Whenever the edges of a complete graph on $n$ vertices are coloured with $r$ colours, on how many vertices can we find a monochromatic, connected subgraph? In this talk, we consider this Ramsey theory type problem and its variants, such as replacing "connected" with "$k$-connected"; replacing "monochromatic" with "$s$-coloured", for some $s \leq r$; and replacing "complete graph" with some other graph. We shall report on some very recent developments on these problems. For example, we shall discuss the current status of a 20 year old conjecture of Bollobás and Gyárfás, which concerns finding a monochromatic $k$-connected subgraph in a 2-edge-coloured complete graph. Many open questions will be presented.

## Extremal numbers and Sidorenko's conjecture

Joonkyung Lee

Yonsei University, Korea

Sidorenko's conjecture states that, for all bipartite graphs $H$, quasirandom graphs contain asymptotically the minimum number of copies of $H$ taken over all graphs with the same order and edge density. While still open for graphs, the analogous statement is known to be false for hypergraphs. We show that there is some advantage in this, in that if Sidorenko's conjecture does not hold for a particular $r$-partite $r$-uniform hypergraph $H$, then it is possible to improve the standard lower bound, coming from the probabilistic deletion method, for its extremal number $\mathrm{ex}(n, H)$, the maximum number of edges in an $n$-vertex $H$-free $r$-uniform hypergraph. With this application in mind, we find a range of new counterexamples to the conjecture for hypergraphs, including all linear hypergraphs containing a loose triangle and all 3-partite 3-uniform tight cycles.

## Degree conditions for the existence of a homeomorphically irreducible spanning tree

Shoichi Tsuchiya

Senshu University, Japan

For a graph $G$ and a spanning tree $T$ of $G$, if $T$ has no vertices of degree 2, then $T$ is called a *homeomorphically irreducible spanning tree* (*HIST*) of $G$. Let $G$ be a connected graph of order $n$. We let $\delta(G)$ denote the *minimum degree* of $G$. We let

$$\sigma_2(G) = \min\{d_G(u) + d_G(v) : u, v \in V(G),\ u \neq v,\ uv \notin E(G)\}$$

if $G$ is not complete; we let $\sigma_2(G) = \infty$ if $G$ is complete.

In 1990, Albertson, Berman, Hutchinson and Thomassen proved that if $\delta(G) \geq 4\sqrt{2n}$, then $G$ has a HIST. In 2022, Ito and Tsuchiya proved that if $\sigma_2(G) \geq n-1$ and $n \geq 8$, then $G$ has a HIST. Recently, we improve a minimum degree condition for a HIST, and refine a degree-sum condition for a HIST.

Although degree-sum conditions for HIST implies minimum degree conditions for a HIST, they are much bigger than $4\sqrt{2n}$. So, we introduce a degree-product condition, which seems to be more reasonable for the existence of a HIST. We let

$$\pi_2(G) = \min\{d_G(u)d_G(v) : u, v \in V(G),\ u \neq v,\ uv \notin E(G)\}$$

if $G$ is not complete; we let $\pi_2(G) = \infty$ if $G$ is complete.

We prove that if $\pi_2(G) \geq cn$ for a constant $c$ and $n \geq 4$, then $G$ has a HIST. This result implies a minimum degree condition for a HIST which is close to $4\sqrt{2n}$.

This is a joint work with Michitaka Furuya (Kitasato University) and Akira Saito (Nihon University).

# Complexity of null dynamical systems and Sauer-Shelah lemmas

Tuan Tran

University of Science and Technology of China, China

The topological entropy of a topological dynamical system, introduced in a foundational paper by Adler, Konheim and McAndrew (1965) is a nonnegative number that measures the uncertainty or disorder of the system. Comparing with positive entropy systems, zero entropy systems are much less understood. In order to distinguish between zero entropy systems, Huang and Ye (2009) introduced the concept of maximal pattern entropy of a topological dynamical system. At the heart of their analysis is a Sauer-Shelah type lemma. In the present paper, we provide a shorter and more conceptual proof of a strengthening of this lemma, and discuss its surprising connection between dynamical system, combinatorics and a recent breakthrough in communication complexity. We also improve one of the main results of Huang and Ye on the maximal pattern entropy of zero-dimensional systems, by proving a new Sauer-Shelah type lemma, which unifies and enhances various extremal results on VC-dimension, Natarajan dimension and Steele dimension. This is joint work with Guorong Gao, Jie Ma and Mingyuan Rong.

**Restaurant Lis:**

- **Itoman Gyomin Shokudo**: Traditional Okinawa food. Fishes. 20min on foot.
- **Itoman Osakana Center**: Fisherman's market. 20min on foot.
- **Nikugoden Itoman Honten**: Japanese Meet BBQ. 25min on foot.
- **Sushiro**: Reasonable Sushi restaurant. 25min on foot.
- **Restaurants inside the hotel**: Find the information in the hotel. 0min on foot.