Distinguishing numbers of 4-regular quadrangulations on the Klein bottle

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The *distinguishing number* is a combinatorial invariant defined for an abstract graph, concerning the symmetry of graphs as follows. Let G be a graph and $c : V(G) \to \{1, 2, ..., d\}$ an assignment of labels to the vertices of G. Such a labeling c is called a *d-distinguishing* labeling of G if no automorphism of G other than the identity map preserves the labels given by c. A graph G is said to be *d-distinguishable* if G admits a d-distinguishing labeling. The distinguishing number of G is defined as the minimum number d such that G is d-distinguishable and is denoted by $D(G)$. For examples, the distinguishing number of a complete graph K_n is $D(K_n) = n$.

Negami and Fukuda have found the distinguishing number of 4-regular quadrangulation on the torus in [2]. Furthermore, Negami [1] has established a general theorem on the distinguishing number of graphs embedded on closed surfaces, using some technique in topological graph theory. In his theory, "the faithfulness of embedding", defined in [1], plays an important role. Acutually, he has proved that polyhedral graphs faithfully embedded on a closed surface are 2-distinguishable with finitely many exceptions.

We shall focus on 4-regular quadrangulatoin on the Klein bottle in turn and prove the following theorem:

Theorem 1. *Every 4-regular quadrangulatoin on the Klein bottle is 2-distinguishable unless it is isomorphic to one of $Q_l(4, 2)$ and $Q_m(2, r)$ with $r \geq 3$.*

There have been classified the 4-regular quadrangulations on the Klein bottle with their standard forms in [3]. According to the classification, any 4-regular quadrangulatoin on the Klein bottle is isomorphic to grid type $Q_g(p, r)$ or ladder type $Q_l(2r, s)$ or mesh type $Q_m(p, r)$ with suitable parameters p, r or s. The exceptional case $Q_l(4, 2)$ and $Q_m(2, r)$ with $r \geq 3$ are ladder type and mesh type of 4-regular quadrangulations on the Klein bottle, respectively.

We prove the distinguishing number of 4-regular quadrangulations on the Klein bottle in [4].

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In this talk, we will show the distinguish numbers of exceptional cases. For examples, Figure 1 shows $Q_l(4,2)$ and one of the distinguishing labelings with distinguishing number three.

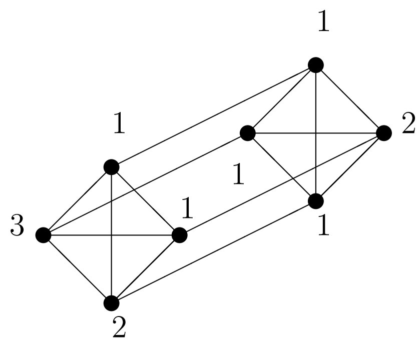

Figure 1 $Q_l(4,2)$ and a labeling

The exception $Q_m(2,4)$ is isomorphic to $K_{4,4}$ and the distinguishing number $D(K_{4,4}) = 5$. The distinguishing number of $Q_l(4,2)$ is equal to 3. The infinite series of those quadrangulations on Klein bottle $Q_m(2,r)$ with $r \geq 3$ have the distinguishing number 3 in Figure 2.

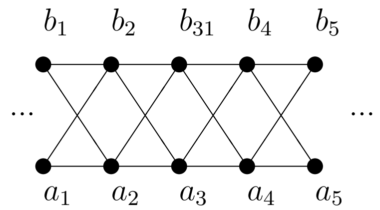

Figure 2 An infinite series of $Q_m(2,r)$ with $r \geq 3$

Consider the auto-homeomorphism of the Klein bottle, then we obtain the following corollary.

Corollary 2. *The distinguishing number of* 4*-regular quadrangulations on the Klein bottle takes only three values* 2*,* 3 *and* 5*.*

References

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