On The Convex Division

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Let V is a finite subset of $\mathbb{R}^2$. We assume that V is in the general position, that is, there are no three points in V which belong to a single line. We denote the convex hull of V by $\mathrm{CH}(V)$. An inner point of V is a point of V which is in the interior of $\mathrm{CH}(V)$ and a boundary point of V is a point of V which is in the boundary of $\mathrm{CH}(V)$. Denote the set of inner point of V by $I(V)$ and the set of boundary point of V by $B(V)$. For a boundary point v of V, let e_1 and e_2 be the two edge on the boundary $\mathrm{CH}(V)$ incident to v, then we call the area between two extended straight lines of e_1 and e_2 the opposite angle area of v of $\mathrm{CH}(V)$ and denote by $OA_{\mathrm{CH}(V)}(v)$ (see Figure 1).

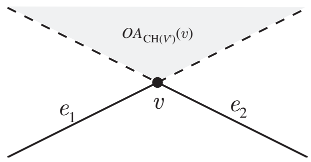

Figure 1 the opposite angle area

Let G be a plane graph G. We call the unbounded region of G the outer region and call the other regions the inner regions. We denote the set of inner regions by $IR(G)$. We call an edge e of G a boundary edge, if e incidents to the outer region and denote the set of boundary edge by $BE(G)$.

A convex division of $\mathrm{CH}(V)$ with respect to V is a plane graph G with vertex set V whose inner regions are convex and $\bigcup IR(G) = \mathrm{CH}(V)$. We simply say a convex division of V, for a convex division of $\mathrm{CH}(V)$ with respect to V. We say that G is a minimal convex division of V, if there are no convex divisions of V with less inner regions than G.

For a region r of a convex division G and for a vertex v of r, we denote by $\angle_r(v)$ the angle between two edges which incident to both of v and r. Let $e = (u, v)$ be an edge of a convex division G and let r_1 and r_2 be the regions of each side of e. We say that v is a convex end of e if $\angle_{r_1}(v) + \angle_{r_2}(v) \leq \pi$. We say that e is an eliminatable edge if u and v are convex ends of e. If e is an eliminatable edge of a convex division G, $G - e$ is a convex division. If $e_1, \dots, e_k$ are some eliminatable edges of a convex division G which are mutually non-adjacent, then $G - \{e_1, \dots, e_k\}$ is a convex division.

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Theorem 1. *Let V is a finite subset of $\mathbb{R}^2$ with i inner points. Then* $\mathrm{CH}(V)$ *has a convex division with respect to V which has less than or equal to* $\left\lfloor \frac{3i+3}{2} \right\rfloor$ *inner regions.*

Proposition 2. *For any non-negative integer i, there is a finite subset V of $\mathbb{R}^2$ with i inner points such that any convex division of* $\mathrm{CH}(V)$ *with respect to V has at least* $\left\lfloor \frac{3i+3}{2} \right\rfloor$ *inner regions.*

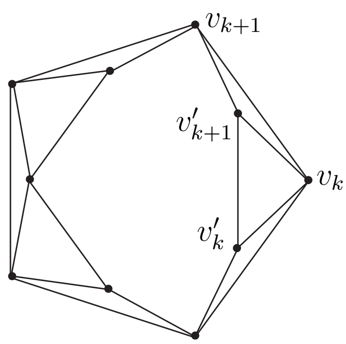

Figure 2 An examle which attains the upper bound of the theorem.