Path Transferability of Graphs on surfaces

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The graphs discussed here are simple and connected. A *path* consists of distinct vertices $v_0, v_1, \ldots, v_n$ and edges $v_0v_1, v_1v_2, \ldots, v_{n-1}v_n$. A path of length n is called an *n-path*. When the direction of the path P needs to be emphasized, it is denoted $\langle P \rangle$. If there is no danger of confusion, we use the same notation P instead of $\langle P \rangle$. We denote the reverse path of P by P^{-1}. The last(resp. first) vertex of a path P in its direction is called the *head* (resp. *tail*) of P and is denoted by $h(P)$(resp. $t(P)$); for $P = \langle v_0v_1 \ldots v_{n-1}v_n \rangle$, we set $h(P) = v_n$ and $t(P) = v_0$. We consider a path as an ordered sequence of distinct vertices with a head and a tail.

A *transfer-move* of a path P is to remove the tail and add a vertex at the head: Let P be an n-path. A new n-path P' is obtained by deleting the vertex $t(P)$ from P and adding v to P as its new head ,(it seems that P takes one step and reaches the next position P'). We say that P *can transfer* (*or move*) *to* P' *by a step* and denote it by $P \to P'$. If there is a sequence of paths $P \to \cdots \to Q$ for two paths P and Q, then we say that P *can transfer* (*or move*) *to* Q, and denote it by $P \dashrightarrow Q$.

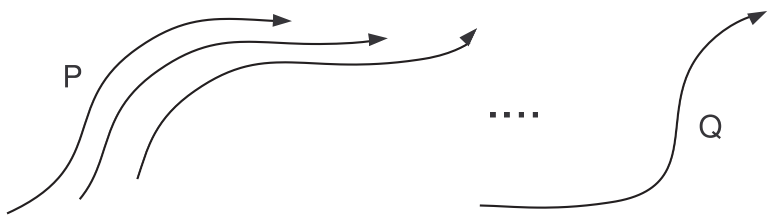

Figure 1

Proposition 1. *Let P, Q be n-paths in a graph. If $P \dashrightarrow Q$, then $Q^{-1} \dashrightarrow P^{-1}$.*

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We regard a path as a "train" that moves along a graph. The main question we study is whether a path can transfer to everywhere on the graph by several steps:

- A graph G is called *n-path-transferable* or *n-transferable* if G contains at least one n-path and if any two n-paths in G can transfer from one to another by finite number of steps, that is, $P \dashrightarrow Q$ holds for any pair of directed n-paths P, Q in G.

- An n-path P in a graph is called *reversible* if P can transfer to P^{-1}. A graph G is called *n-path-reversible* or *n-reversible* if G contains at least one n-path and if any n-path in G is reversible.

We showed in [2], [3] the followings:

Proposition 2. ([2]) *If a graph G is n-transferable, then G is $(n-1)$-transferable.*

Theorem 3. ([2]) *A graph G is n-transferable if and only if G is n-reversible.*

Theorem 4. ([3]) *Unless it is complete or a cycle, a connected graph is δ-transferable, where $\delta \geq 2$ is the minimum degree.*

The maximum number n for which a graph is n-transferable is called its *path transferability.*

Using the result of S. Jendrol' and Z. Skupień [1], we further showed the following result for planar graphs.

Theorem 5. ([4]) *Every planar graph with minimum degree at least three has transferability ≤ 10.*

In this talk I will describe recent study of path transferability of graphs on other surfaces.

References

[1] S. Jendrol', Z. Skupień, Local structures in plane maps and distance colourings, *Discrete Math.* **236** (2001), 167-177.

[2] R. Torii, Path transferability of graphs, *Discrete Math.* **308** (2008), 3782-3804.

[3] R. Torii, Path transferability of graphs with bounded minimum degree, *Discrete Math.*, to appear.

[4] R. Torii, Path transferability of planar graphs, submitted.