The Distinguishing Numbers of Triangulations on the Torus

Terukazu Sano*

A graph G is said to be *d-distinguishable* if there exists an assignment $f\colon V(G) \to \{1, 2, \cdots, d\}$ of distinct labels to its vertices, such that if no automorphism of G other than the identity map preserves the labels given by f. The *distinguishing number* of G is defined as the minimum number d such that G is d-distinguishable and is denoted by $D(G)$.

There are several studies for the distinguishing numbers of some classes which consist of abstruct graphs. On toporogical graph theory, Tucker has established the distinguishing numbers of maps on surfaces [1]. Fukuda, Negami and Tucker [2] have proved that every triangulation on the sphere is 2-distinguishable, except K_4, $K_{2,2,2}$, $\overline{K_2} + C_3$ and $\overline{K_2} + C_5$. Negami has showed that all triangulations "faithfully embedded" on a fixed surface are 2-distinguishable with finitely many exceptions [3]. These mainly discussed the 2-distinguishability of graphs on surfaces.

Generally, some graphs on a fixed surface may not be 2-distinguishable. Accordingly, determing the distinguishing numbers of graphs on surfaces is beneficial to studying distinguishability of graphs. The distinguishing number of triangulations G on the projective plane has been determined as follows:

Theorem 1. (Negami [5]) *The distinguishing number of triangulations G on the projective plane takes only five values* 1, 2, 3, 4 *and* 6.

- $D(G) = 3$ *if and only if G is isomorphic to either $K_{3,3,3} - K_{3,3}$ or to $H + K_3$ for a maximal planar graph H with at least four vertices.*
- $D(G) = 4$ *if and only if G is isomorphic to $K_4 + \overline{K_3}$.*
- $D(G) = 6$ *if and only if G is isomorphic to K_6.*

Similar to this theorem, we shall determine the distinguishing numbers of triangulations on the torus. Our main result is the following:

Theorem 2. *The distinguishing number of triangulations G on the torus takes only five values* 1, 2, 3, 4 *and* 7.

- $D(G) = 3$ *if and only if G is isomorphic to* C9*, F10*, G13, G14, J4, J5, L, T8, T20 *or* T21.
- $D(G) = 4$ *if and only if G is isomorphic to* H8, T2, T5 *or* T7.
- $D(G) = 7$ *if and only if G is isomorphic to K_7.*

*Graduate School of Environment and InformationSciences, Yokohama National University. E-mail: d08tc004@ynu.ac.jp

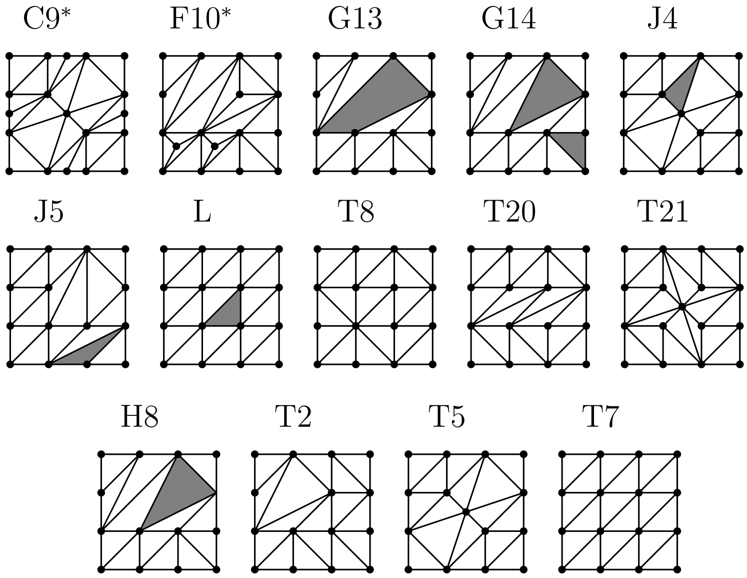

Figure 1

To get the actual triangulations in Figure 1, we need to identify the two horizontal segments, and two vertical segments, and subdivide shaded regions suitably.

The theorem tells us two observations: (i) there are infinitely many triangulations which is not 3-distinguishable, and (ii) every 5-triangulation on the torus is 2-distinguishable, except K_7, $K_{2,2,2,2}$, T8 and T20.

In our talk, we introduce the theory of "re-embedding structure" for triangulations on surfaces and how to apply the theory to the study for distiguishing numbers of triangulations on closed surfaces.

References

[1] T. W. Tucker, Distinguishability of maps, preprint 2005.

[2] T. Fukuda, S. Negami and T. Tucker, 3-Connected planar graphs are 2-distinguishable with few exceptions, to appear in *Yokohama Math. J.* (2008).

[3] S. Negami, The distinguishing number of graphs on closed surfaces, preprint in 2006.

[4] A. Nakamoto, S. Negami and T. Tanuma, Re-embedding structures of triangulations on closed surfaces, *Sci. Rep. Yokohama Nat. Univ., Sec. I* **44** (1997), 41–55.

[5] S. Negami, The distinguishing numbers of triangulations on the projective lane, *Congressus Numernatium* **186** (2007), 135–144.

[6] A. Sasao, Panel structures of triangulations on the torus, *Discrete Math.* **303** (2005), no. 1-3, 186–208.