On Chromatic Uniqueness of a Family of K_4-Homeomorphs

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All graphs considered here are simple graphs. For such a graph G, let $P(G,\lambda)$ (simply $P(G)$) denote the chromatic polynomial of G. Two graphs G and H are chromatically equivalent (or simply $\chi-$equivalent), denoted by $G \sim H$, if $P(G,l) = P(H,l)$ (simply $P(G) = P(H)$). A graph G is chromatically unique (or simply $\chi-$unique) if for any graph H such that $H \sim G$, we have $H \cong G$, i.e, H is isomorphic to G.

A K_4-homeomorph is a subdivision of the complete graph K_4. Such a homeomorph is denoted by $K_4(a,b,c,d,e,f)$ if the six edges of K_4 are replaced by the six paths of length a,b,c,d,e,f, respectively. So far, the chromaticity of K_4-homeomorphs with girth g, where $3 \le g \le 9$ has been studied by many authors (see [1,2,3,4]). Recently, Peng in [5] has studied the chromaticity of one type of K_4-homeomorphs with girth 7, that is the chromaticity of $K_4(1,3,3,d,e,f)$. In the whole study of K_4-homeomorphs with girth 10, we need to consider 24 types of K_4-homeomorphs. In this paper, we discuss the chromaticity of one of these types, namely $K_4(3,3,4,d,e,f)$, where d,e,f are at least 3. The chromaticity of the other types of K_4-homeomorphs with girth 10 will be presented in other papers. We also study the chromaticity of $K_4(a,a,a+1,d,e,f)$ where min $\{d,e,f\} \ge a$ and $a \ge 3$.

Lemma 1. *Let $G \cong K_4(3,3,4,d,e,f)$ and $H \cong K_4(3,3,4,d',e',f')$, then*

(1) $P(G) = (-1)^{x-1}[s/(s-1)^2][-s^{x-1} - s^5 - 3s^4 - 2s^3 + s^2 + 3s + 2 + R(G)]$, *where* $R(G) = -s^d - s^e - s^f - s^{d+1} - s^{e+1} - s^{f+1} + s^{d+3} + s^{f+3} + s^{e+4} + s^{e+6} + s^{d+7} + s^{f+7} + s^{d+e+f}$, $s = 1-\lambda$, *x is the number of the edges of G.*

(2) *If $P(G) = P(H)$, then $R(G) = R(H)$.*

Our main results are the following:

Theorem 2. *K_4-homeomorph $K_4(3,3,4,d,e,f)$ with girth 10, where d,e,f are at least 3, is $\chi-$unique.*

Theorem 3. *K_4-homeomorph $K_4(a,a,a+1,d,e,f)$ with girth $3a+1$, where min $\{d,e,f\} \ge a$ and $a \ge 3$ is $\chi-$unique.*

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