Spanning Trees with Bounded Epsilon Excess

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We talk about spanning trees. We give sufficient conditions for a graph to have a spanning tree with bounded epsilon excess, in terms of the number of components obtained when we delete a set of vertices. For a spanning subgraph H of a connected graph G, and n is a nonnegative integer, the n-excess of a vertex v is defined to be $\max\{0, \deg_H(v)-n\}$. The total n-excess $te(H,n)$ is the summation of the n-excesses of all vertices, namely,

$$te(H,n) = \textstyle\sum_{v\in V(H)} \max\{0, \deg_H(v) - n\}.$$

Let n be an integer more than 3 and ϵ be a nonnegative number smaller than 1. Let S be a subset of $V(G)$ and $\omega(G \setminus S)$ denote the number of components of a graph $G \setminus S$.

Theorem 1. *If for any S,*

$$\frac{|S|}{\omega(G-S)} \geq \frac{1-\epsilon}{n-2+\epsilon}$$

holds, then there exists a spanning tree T such that

$$te(T,n) \leq \epsilon|V(G)| - 2.$$

Like this, when the total excess bounds with epsilon, we call this *epsilon excess*.

First, when we substitute ϵ for 0. We get this.

Corollary 2. *If for any S,*

$$\frac{|S|}{\omega(G-S)} \geq \frac{1-\epsilon}{(n+1)-2+\epsilon} = \frac{1}{n-1}$$

holds, then there exists $(n+1)$-tree. That is a spanning tree T such that

$$te(T,n) \leq \frac{1}{n}(|V(G)|-2).$$

Second, when we substitute ϵ for $\frac{1}{n}$. We get this.

Corollary 3. *If for any S,*

$$\frac{|S|}{\omega(G-S)} \geq \frac{1-\epsilon}{n-2+\epsilon} = \frac{1}{n-1}$$

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holds, then there exists a spanning tree T such that

$$te(T,n) \leq \frac{1}{n}|V(G)| - 2.$$

We gave same condition for two, but their conclusions are not same.

We study them and we found that they are essentially equivalent. That is to say.

THEOREM 4. *If for any S,*

$$\frac{|S|}{\omega(G-S)} \geq \frac{1}{n-1}$$

holds, then there exists $(n+1)$-tree T such that

$$te(T,n) \leq \frac{1}{n}|V(G)| - 2.$$

References

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