Domination tricritical graph

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The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set of G, and a dominating set of minimum cardinality is called a $\gamma(G)$-set. Note that removing a vertex can increase the domination number by more than one, but can decrease it by at most one. A graph G is a critical (bicritical) graph if the removal of any vertex (pair of vertices) decreases the domination number. For many graphs parameters, criticality (bicriticality) is a fundamental question. Much has been written about those graphs where a parameter goes up or down whenever an edge or a vertex (two vertices) is removed (are removed). Here, the tricritical graphs are introduced and those graphs where the domination number decreases on the removal of any set of 3 vertices are studied, we show with tricritical. It is useful to write the vertex set of a graph as a disjoint union of three sets according to how their removal affects $\gamma(G)$. Let $V(G) = V^0 \cup V^+ \cup V^-$ where $V^0 = \{v \in V|\gamma(G-v) = \gamma(G)\}, V^+ = \{v \in V|\gamma(G-v) > \gamma(G)\}, V^- = \{v \in V|\gamma(G-v) < \gamma(G)\}$, see[1,2,3]. We denote the distance between two vertices x and y in G by $d_G(x,y)$ and the diameter of G denoted by $diam(G)$, is the maximum $d_G(x,y)$ for $\{x,y\} \subseteq V(G)$, see[4].

Observation 1 *What relation does exist between criticality, bicriticality and tricricality of a graph?*

Observation 2 *If G is a connected tricritical graph such that $diam(G) = 2$ then $\forall\ x,y,z \in V(G),\ \gamma(G-\{x,y,z\}) \geq \gamma(G) - 2$.*

Observation 2 implies that, if $\gamma(G-\{x,y,z\}) = \gamma(G) - 3$ for any three distinct vertices x,y and z, then G has no edge.

Observation 3 *The tricritical graph has no a vertex of degree 3.*

Proposition 4 *If G is a tricritical graph, then $V = V^- \cup V^0$, that is, $V^+ = \emptyset$. Furthermore, (1) either G is critical, or $G - v$ is bicritical for all $v \in V^0$ and (2) either G is bicritical or $G - \{v,w\}$ is critical for every $\{v,w\}$ such that $\gamma(G-\{v,w\}) = \gamma(G)$.*

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2-tricritical graphs is characterized.

Observation 5 *There is not any* 2*-tricritical graph with* 5 *vertices.*

Proposition 6 *A graph* G *is* 2*-tricritical if and only if* $G = P_1 \cup K_1$, P_3, $2P_1$, $P_2 \cup K_1$, $C_3 \cup K_1$ *or* $G = K_{2n} - M$ *where* $n \geq 2$ *and* M *is a perfect matching in* K_{2n}.

Proposition 7 *Let* G *be a connected tricritical graph, then we have:*
1. If G *is bicritical, then* $\delta(G) \geq 4$.
2. If G *is not bicritical, then* $\delta(G) = 2$ *or* $\delta(G) \geq 4$.

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