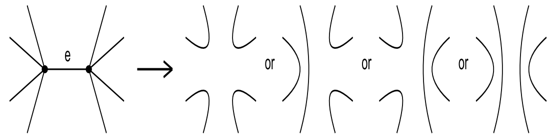
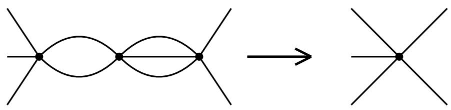
Generating 5-regular planar graphs

Guoli Ding [*], Jinko Kanno and Jianning Su [†]

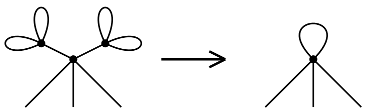
For $k = 0, 1, 2, 3, 4, 5$, let $\mathcal{P}_k$ be the class of k-edge-connected 5-regular planar graphs. In this paper, graph operations are introduced that generate all graphs in each $\mathcal{P}_k$.



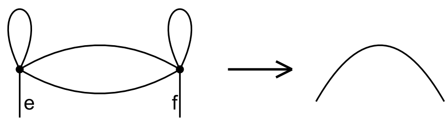
Operation $\mathcal{O}_1$.



Operation $\mathcal{O}_3$.



Operation $\mathcal{O}_4$.



Operation $\mathcal{O}_5$.

Theorem 1. *Every graph in $\mathcal{P}_0$ can be reduced within $\mathcal{P}_0$ by $\mathcal{O}_1$ and $\mathcal{O}_3$ to a graph for which every component is $5K_2, 3K_2^L$ or K_2^{2L}.*

Theorem 2. *Every graph G in $\mathcal{P}_1$ can be reduced within $\mathcal{P}_1$*

(i) *to $5K_2$, $3K_2^L$ or K_2^{2L} by $\mathcal{O}_1$, $\mathcal{O}_3$, and $\mathcal{O}_4$; and*
(ii) *to $3K_2^L$ or K_2^{2L} by $\mathcal{O}_1$, $\mathcal{O}_2$, and $\mathcal{O}_4$, unless $G = 5K_2$.*

Theorem 3. *Every graph in $\mathcal{P}_2$ can be reduced within $\mathcal{P}_2$ by $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$, $\mathcal{O}_5$, and $\mathcal{O}_6$ to $3K_2^L$ or $5K_2$.*

Theorem 4. *Every graph in $\mathcal{P}_3$ can be reduced within $\mathcal{P}_3$ by $\mathcal{O}_1, \mathcal{O}_3, \mathcal{O}_7$, and $\mathcal{O}_8$ to $5K_2$ or $3K_2^L$.*

Theorem 5. *Every graph in $\mathcal{P}_4$ can be reduced within $\mathcal{P}_4$ by $\mathcal{O}_1, \mathcal{O}_9$, and $\mathcal{O}_{10}$ to $5K_2$.*

[*]Department of Mathematics, Louisiana State University, Baton Rouge, Louisiana 70803, USA. E-mail: `ding@math.lsu.edu`

[†]Mathematics and Statistics Program, Louisiana Tech University, Ruston, Louisiana 71272, USA. E-mail: `jkanno, jsu007@latech.edu`

THEOREM 6. *Every graph in $\mathcal{P}_5$ can be reduced within $\mathcal{P}_5$ by $\mathcal{O}_1$, $\mathcal{O}_{11}$, and $\mathcal{O}_{12}$ to $5K_2$.*

Class	Operation	Minimum graphs
$\mathcal{P}_0$	$\mathcal{O}_1, \mathcal{O}_3$	$5K_2, 3K_2^L, K_2^{2L}$
$\mathcal{P}_1$	$\mathcal{O}_1, \mathcal{O}_3, \mathcal{O}_4$	$5K_2, 3K_2^L, K_2^{2L}$
$\mathcal{P}_1$	$\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_4$	$5K_2, 3K_2^L, K_2^{2L}$
$\mathcal{P}_2$	$\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_5$	$5K_2, 3K_2^L, K_2^{2L}$
$\mathcal{P}_3$	$\mathcal{O}_1, \mathcal{O}_3, \mathcal{O}_7, \mathcal{O}_8$	$5K_2, 3K_2^L$
$\mathcal{P}_4$	$\mathcal{O}_1, \mathcal{O}_9, \mathcal{O}_{10}$	$5K_2$
$\mathcal{P}_5$	$\mathcal{O}_1, \mathcal{O}_{11}, \mathcal{O}_{12}$	$5K_2$

Further Remarks

For $k = 1, 2, 3, 4, 5$, we proved generating theorems for the classes $\mathcal{P}_k$ of all k-edge-connected 5-regular planar graphs.

One of next natural questions is to pursue *splitter theorems* for $\mathcal{P}_k$ and $\mathcal{P}_{k,g}$. Suppose a graph G "contains" another graph H. Then how can G be built up from H in such a way that certain properties of G and H are preserved during the construction process? Probably the best-known result to answer this kind of question is the one by Seymour [7], for general matroids, and Negami [6], for graphs only, which explains the construction when the containment relation is the minor relation and the property to preserve is 3-connectedness. These answers are known as *splitter theorems*. Ding and Kanno have proved splitter theorems for several classes of 3-regular, 4-regular or 4-regular planar graphs (see [2, 3, 4]).

References

[1] J. W. Butler, A generation procedure for the simple 3-polytopes with cyclically 5-connected graphs, *Canadian Journal of Mathematics* **XXVI** (1974), No.3, 686-708.

[2] G. Ding and J. Kanno, Splitter Theorems for cubic graphs, *Combin. Probab. Comput.***15** No.3 (2006), 355-375.

[3] G. Ding and J. Kanno, Splitter Theorems for 4-regular graphs, submitted.

[4] J. Kanno, *Splitter theorems for* 3- *and* 4-*regular graphs*, Ph.D. dissertation, Louisiana State University, Baton Rouge, Louisiana, 2003.

[5] J. Kanno and M. Kriesell, A generating theorem for 5-regular simple planar graphs I, *Congressus Numerantium* **185** (2007), 127-143.

[6] S. Negami, A characterization of 3-connected graphs containing a given graph, *J. Combinatorial Theory* Series B, **32** (1982), 69-74.

[7] P. D. Seymour, Decomposition of regular matroids, *J. Combinatorial Theory* Series B, **28** (1980), 305-359.

[8] A. Schrijver, *Combinatorial Optimization: Polyhedra and Efficiency*, Springer, Berlin, 2003