Some results concerning non-separating subgraphs in k-connected graphs

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1. Introduction

All graphs considered here are finite, undirected, and without loops or multiple edges. We report some recent results on non-separating subgraphs in highly connected graphs. Here *a non-separating subgraph* denotes a subgraph whose deletion keeps high connectivity. Note that if we delete a contractible edge in a k-connected graph, the resulting graph will be $(k-1)$-connected. Hence, the existence of a contractible subgraph shows the existence of a non-separating subgaph at the same time. In this talk, we focus on the existence of non-separating subgraphs in highly connected graphs, and introduce some related results.

2. Degree conditions for removable edges

The old well-known result of Chartrand, Kaugars and Lick [1] says that every k-connected graph G with minimum degree at least $3k/2$ has a vertex v such that $G-v$ is still k-connected. This theorem tells us that if we want to find a vertex v in a k-connected graph G such that $G-v$ is still k-connected, then the minimum degree $3k/2$ is enough. But what if we want to find an edge e such that $G-V(e)$ is still k-connected ? What minimum degree condition is necessary ? Motivated by this question, we shall prove the following result.

Theorem 2.1 *Let k be an integer with $k \geq 2$. Suppose G is a k-connected graph with minimum degree at least $\lfloor 3k/2 \rfloor + 2$. Then G has an edge e such that $G-V(e)$ is still k-connected.*

3. Contractible triples

It is well known that every 3-connected graph of order 5 or more has an edge whose contraction still results in a 3-connected graph (see [10]). McCuaig and Ota [8] has extended this result by showing that every 3-connected graph with at least 9

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vertices has a connected graph of order 3 whose contraction results in a 3-connected graph. This result was further extended by Kriesell [7].

Our purpose is to extend these results to a higher connected graph. But, as Thomassen [9] pointed out, there exist infinitely many k-connected k-regular graphs which do not have an edge whose contraction results in a k-connected graph for $k \geq 4$. Therefore, we have to set a modest goal: we would like to consider a k-connected graph which has either some fixed subgraph or a connected subgraph of order 3 whose contraction results in a k-connected graph. Let us remind that Thomassen [9] proved that every k-connected graph has either a contractible edge or a triangle. This result was further extended by Kawarabayashi [6]. There it is proved that every k-connected graph contains either a contractible maximal clique with order at most 3 or a K_4^-. However, in order to achieve our goal, a triangle or a K_4^- does not seem enough to exclude. Our result is the following:

Theorem 3.1 *Let k be an integer with $k \geq 2$. If G is k-connected, then G contains either C_4 or a connected subgraph of order* 3 *whose deletion results in a $(k-1)$-connected graph.*

In this talk, we will mention about the details of the above results and also we will further report some other latest results in [2, 3, 4, 5].

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